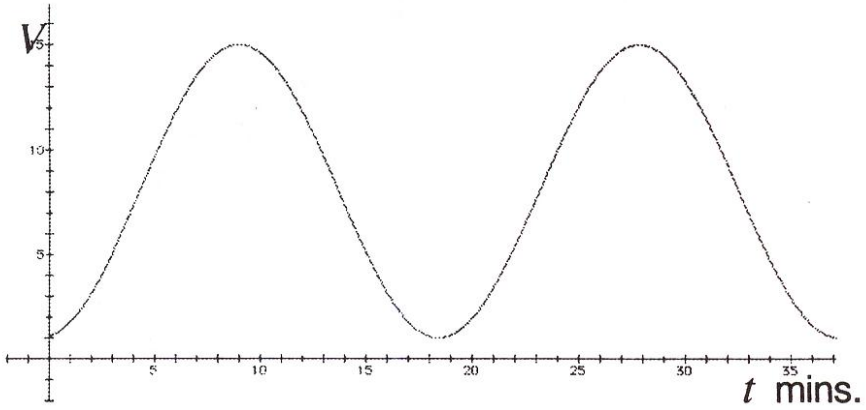
TRIG APPLICATIONS

Example 1: The university has a local radio station which broadcasts throughout the Student Union Building. Students are concerned about the erratic volume levels. A student takes meter readings of the volume levels and discovers that the volume follows a pattern modelled by the equation:

$V = 7\cos(\frac{t}{3} - 3) + 8$ where V = volume, and t = time in minutes.



a) How many minutes pass before the highest volume and the lowest?

The period T is the time for one cycle i.e. time between highest volume and next highest volume. So the time between the highest volume and the next lowest is half the period. $T = \frac{2\pi}{B} = \frac{2\pi}{\frac{1}{3}} = 6\pi$. Therefore the time between the highest volume and the next lowest is half of this, $3\pi = 9.425 = 9$ minutes, 25 seconds.

b) How many minutes pass before the student first has a reading of 12 units?

Solve $V = 12$ i.e. $7\cos(\frac{t}{3} - 3) + 8 = 12.$ Solve this on your graphics calculator:
In Graphs, type in:

- $y_1 = 7\cos(\frac{t}{3} - 3) + 8$
- $y_2 = 12$

Set Angle mode to Radians.

$A = 7,\ B = \frac{1}{3},\ C = -9,\ D = 8$ } C is -9 and **NOT** -3, because the correct(factorised) form of the equation is $7\cos\left[\frac{1}{3}(t - 9)\right] + 8$.

Viewing Window Settings

$x_{min} = 0$
$x_{max} = 12\pi$
$y_{min} = 1$
$y_{max} = 15$

} x_{min} and x_{max} values are determined by the range in which you want to find the solutions. In this case, the diagram is showing 2 cycles so x_{max} should be double the period, i.e. 12π.

Once both graphs are displayed, press F5 for G-Solv, press F5 for ISCT, then wait for the graphics calculator to display the first solution: $x = 6.11$, i.e. 6 minutes 7 seconds. This is how long it takes to get the first reading of 12 units.

c) For what percentage of the first 20 minutes is the volume level above 12 units?

Continuing on from part b), find the next time that the volume level hits 12 units. While still in GRAPH mode with the graph on display and using ISCT to find the first solution, press the right arrow to find the second solution: $x = 11.89$, i.e. 11 minutes 53 seconds.

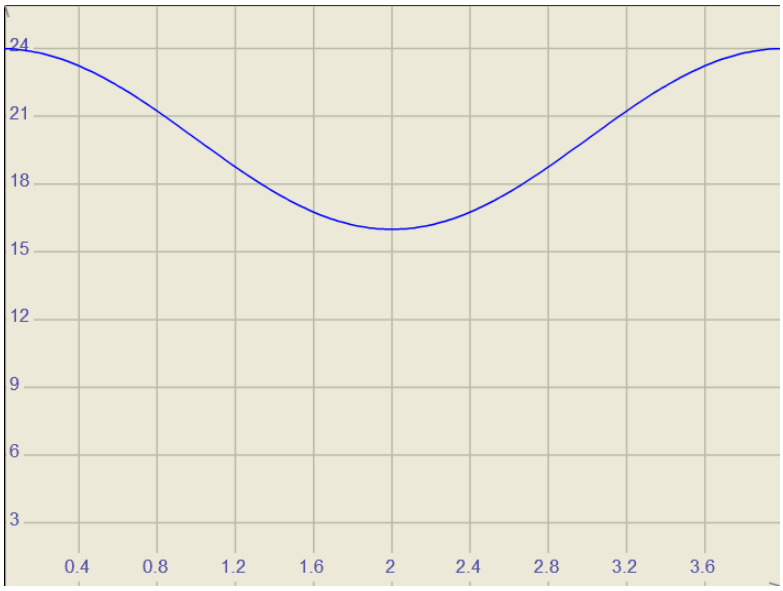
In the first 20 minutes, the volume level is above 12 units between the time 6 minutes 7 seconds and 11 minutes 53 seconds. This is a period of 5 minutes and 47 seconds, out of 20 minutes. This is a percentage of about 29%.

Example 2: Crackit's Extreme Sports Company is trialling a new bungy cord from a bridge across the Clutha River. The safety team tie an object weighing 80 kg to the newly developed cord, drop it over the bridge then measure its height above the water over a period of 20 seconds. The oscillation of the object is modelled by the equation:

$h = 20 + 4\cos(\frac{\pi t}{2})$ where h is the height in metres above the water line,
and t is the time in seconds after the object is dropped.

a) How high is the bridge "drop off point" above the water?

A diagram of the graph would be helpful. Graph of function shown below:



The bridge "drop off point" is the highest (maximum) y value of the graph, which is 24 metres.

b) How long does one complete oscillation take?

The period is $T = \frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{2}} = 4$ seconds, i.e. one oscillation takes 4 seconds.

c) How long after the start does the object first reach 2 metres below the bridge "drop off point"?

2 metres below the bridge "drop off point" is 24 – 2 = 22 metres. So we are solving $h = 22$ i.e. solve $20 + 4\cos(\frac{\pi t}{2}) = 22$. Solve this on your graphics calculator:

In Graphs, type in:

- $y_1 = 20 + 4\cos(\frac{\pi t}{2})$
- $y_2 = 22$

Set Angle mode to Radians.

$A = 4,\ B = \frac{\pi}{2},\ C = 0,\ D = 20$

Viewing Window Settings

$x_{min} = 0$
$x_{max} = 4$
$y_{min} = 16$
$y_{max} = 24$

x_{min} and x_{max} values are determined by the range in which you want to find the solutions. In this case, we only need one cycle i.e. 4.

Once both graphs are displayed, press F5 for G-Solv, press F5 for ISCT, then wait for the graphics calculator to display the first solution: $x = 0.67$ minutes, i.e. 40 seconds. So it takes the object 40 seconds to first reach 2 metres below the bridge "drop off point".

Delta Ex 33.3 pg 311 – 312 Q 1 – 5.